Секция «Вещественный, комплексный и функциональный анализ»

Discrete bilinear Hardy type inequality

Научный руководитель – Темирханова Айнур Мараловна

Sadirova G.A.[1], *Zhangabergenova N.S.*[2]

1 - Евразийский национальный университет имени Л. Н. Гумилёва, Механико-математический факультет, Нур-Султан, Казахстан, *E-mail: gulden.sadirova@mail.ru*; 2 - Евразийский национальный университет имени Л. Н. Гумилёва, Механико-математический факультет, Нур-Султан, Казахстан, *E-mail: zhanabergenova.ns@gmail.com*

Let $1 < p, s, q < \infty$; let $u = \{u_n\}$, $\vartheta = \{\vartheta_n\}$, $\omega = \{\omega_n\}$, $n \in N$ be positive sequences of real numbers. Let $f = \{f_n\}_{n=1}^{\infty}$ $g = \{g_n\}_{n=1}^{\infty}$ be arbitrary sequences of nonnegative numbers. In this work we study the characterization problem for the bilinear discrete Hardy type operators of the following form

$$\left(\sum_{n=1}^{\infty} u_n^q \left(\sum_{i=1}^{n} a_{ni} f_i\right)^q \left(\sum_{i=1}^{n} g_i\right)^q\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty} \vartheta_i^p f_i^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{\infty} \omega_i^s g_i^s\right)^{\frac{1}{s}}, \qquad (1)$$

where C is the best constant in (1) and does not depend on f and g; (a_{ij}) is non-negative matrix with elements $a_{ij} \geq 0$, when $i \geq j \geq 1$, $a_{ij} = 0$, when $i < j$ and which satisfy the Oinarov's condition: there exists $d \geq 1$ such that

$$\frac{1}{d}(a_{ik} + a_{kj}) \leq a_{ij} \leq d(a_{ik} + a_{kj}), \forall i \geq k \geq j \geq 1 \qquad (2)$$

When $a_{i,j} = 1, i \geq j \geq 1$ the inequality (1) was investigated in [1],[2] for various combinations of the parameters p, s and q.

Our main result reads as follows:

Теорема 1. *Let $1 < p, s \leq q < \infty$ and the elements of matrix (a_{ij}) satisfy condition (2). Then the inequality (1) holds if and only if $A = max\{A_1, A_2\} < \infty$, where*

$$A_1 = \sup_{n\geq 1} \left(\sum_{i=1}^{\infty} u_i^q\right)^{\frac{1}{q}} \left(\sum_{j=1}^{n} a_{nj}^{p'} \vartheta_j^{-p'}\right)^{\frac{1}{p'}} \left(\sum_{i=1}^{n} \omega_i^{-s'}\right)^{\frac{1}{s'}} \qquad (3)$$

$$A_2 = \sup_{n\geq 1} \left(\sum_{i=1}^{\infty} a_{in}^q u_i^q\right)^{\frac{1}{q}} \left(\sum_{i=1}^{n} \vartheta_i^{-p'}\right)^{\frac{1}{p'}} \left(\sum_{i=1}^{n} \omega_i^{-s'}\right)^{\frac{1}{s'}} \qquad (4)$$

Moreover, where $A \approx C$ is best constant in (1).

References

1) 1. Jain P., Stepanov V.D., Shambilova G.E. Discrete Bilinear Hardy Inequalities //Dokl. Math.,2019.Vol. 3(100). p. 554-557.
2) 2. Pankaj J., Kanjilal S., Shambilova G.E., Stepanov V.D. Bilinear weighted Hardy-type inequalities in discrete and qq-calculus // Math. Inequal. Appl.,2020. Vol. 4(23). p. 1279-1310.