Секция «Вещественный, комплексный и функциональный анализ»

Weighted estimates for discrete iterated Hardy operator

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Let $0 < r < \infty$ and $0 < p \leq q < \infty$. Let $\{u_i\}_{i=1}^{\infty}$, $\{\vartheta_i\}_{i=1}^{\infty}$, $\{\omega_i\}_{i=1}^{\infty}$ be non-negative sequences of real numbers. Denote by $l_{p,\vartheta}$ a space of all sequences $f = \{f_i\}_{i=1}^{\infty}$ of real numbers such that $\|f\|_{l_{p,\vartheta}} = \left(\sum\limits_{i=1}^{\infty} \vartheta_i |f_i|^p\right)^{\frac{1}{p}}$, where $1 \leq p < \infty$.

We consider the following iterated discrete Hardy inequality

$$\left(\sum_{i=1}^{\infty} u_i \left(\sum_{k=1}^{i} \left|\sum_{j=k}^{i} f_j\right|^r \omega_k\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty} \vartheta_i |f_i|^p\right)^{\frac{1}{p}}, \forall f \in l_{p,\vartheta} \tag{1}$$

Compared to the continuous case the discrete analogue of the Hardy iterated inequality is studied very little (see [1], [2]).

The aim of the this work is to obtain necessary and sufficient conditions for the fulfillment of the discrete iterated Hardy inequality (1) for cases: $0 < p \leq 1$, $0 < p \leq min\{q, r\} < \infty$; $0 < r < 1 < p \leq q < \infty$.

Let

$$U_z^- = \left(\sum_{i=z}^{\infty} u_i\right)^{\frac{1}{q}}, A_{1,z}^- = \sup_{k \in [1,z]} \left(\sum_{j=1}^{k} \omega_j\right)^{\frac{1}{r}} \vartheta_k^{-\frac{1}{p}},$$

$$B_{1,z}^- = \left(\sum_{k=1}^{z} \left(\sum_{j=1}^{k} \omega_j\right)^{\frac{p}{p-r}} \left(\sum_{j=k}^{z} \vartheta_j^{1-p'}\right)^{\frac{p(r-1)}{p-r}} \vartheta_k^{1-p'}\right)^{\frac{p-r}{pr}}.$$

Our main result reads as follows:

Теорема 1. *The inequality (1) holds if and only if*
(i) $E = \sup_{z \in N} U_z^- A_{1,z}^- < \infty$ *for* $0 < p \leq 1$, $0 < p \leq min\{q, r\} < \infty$;
(ii) $E = \sup_{z \in N} U_z^- B_{1,z}^- < \infty$ *for* $0 < r < 1 < p \leq q < \infty$.
Moreover $E \approx C$. *Here,* C *is the best constant in the inequality (1).*

References

1) Gogatishvili A., Krepela M., Rastislav O., Pick L. Weighted inequalities for discrete iterated Hardy operators // 2019. URL: https://arxiv.org/abs/1903.04313 (Submitted on 11 March 2019).

2) Oinarov R., Omarbayeva B.K., Temirkhanova A.M. Discrete iterated Hardy-type inequalities with three weights // Journal of Mathematics, Mechanics, Computer Science, 2020. Vol. 1(105). P. 19-29.