Секция «Дифференциальные уравнения, динамические системы и оптимальное управление»

A numerical resolution method based on the Pontryagin's minimum principle (PMP) : The shooting method.

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We present a numerical resolution method based on the Pontryagin's minimum principle "PMP" which is useful in applications called the shooting method. We know that the PMP only provides the necessary conditions optimality whose formulation involves, as for the Linear Quadratic system. In contrast, the PMP does not say anything about the existence of optimal control or the sufficiency of these conditions. The interest of the PMP practice is to allow us to do a first screening of candidate controls for optimality, hoping that the checks verifying the necessary conditions for optimality of the PMP are not too numerous, we can then examine them individually to determine the optimal character or not.

1- The Pontryagin's minimum principle "PMP" :

We take a non-linear control system. Remember that the dynamic is written in the form
x'u(t) = f(t, x_u(t), u(t)), ∀ t ∈ [0,T], $x_u(0)$ = x0, (1.1)
with T>0, f : [0,T]×R^d×U → R^d and x_0∈R^d
The set of admissible checks is U=L^1([0,T];U), (1.2)
where U is a closed non-empty subset of R^k . The objective is to find an optimal control u'∈U which minimizes the criterion J(u)=$_0{}^T$ g(t,x_u(t), u(t))dt + h(x_u(T)) (1.3)
where the functions **g** and **h**: are given.
The control problem optimal is therefore the following: Find u'∈U such that J(u')=$\inf_{u\in U}$J (u). (1.4)

Theorem 1 (PMP). If u'∈U is an optimal control, i.e., if u' is a solution of (1.4), then by noting x'=$x_{u'}$∈AC([0,T];R^d) the trajectory associated with the control u' and by defining the deputy state p'∈AC([0,T];R^d) solution of :
dp'/dt (t)=−A'(t)†p'(t) - b'(t), ∀ t ∈ [0, T], p'(T) = dh/dx (x'(T)) ∈ R^d , (1.5)
where for all t ∈ [0,T]:
A'(t)=df/dx(t,x'(t),u'(t))∈$R^{d\times d}$, b'(t)=dg/dx(t,x'(t),u'(t))∈R^d , (1.6)

we have p.p. t∈[0,T] : u'(t)∈ $\arg_{v\in U}$min H(t, x'(t), p'(t), v), (1.7)
where the Hamiltonian H: [0, T] × R^d×R^d×U → R is an Hamiltonian defined in the references. A triplet (x',p',u') satisfying the above conditions is called an **extreme**. Note that with the conventions adopted, dg/dx and dh/dx are column vectors.

2. Numerical resolution: The method of shooting

The PMP can be used as a basis for a numerical method of solving the optimal control problem: the shooting method. This method is interesting when it is easy to minimize the Hamiltonian, i.e., when we are able to evaluate a function:

ζ(t, x, p) $\in$ $\mathrm{arg}_{v\in U}$min H(t,x,p,v), ∀ (t,x,p) $\in$ [0,T] $\times$ $R^d \times R^d$. (2.1)
In this case, by setting z(t)=(x (t),p(t))†$\in$ $R^d \times R^d$, we obtain the differential system :
z'(t) = F(t, z(t)), ∀ t $\in$ [0,T], (2.2)
with F = (F_x, F_p): [0, T] $\times$ $R^d \times R^d \rightarrow R^d \times R^d$ such as :
F_x(t,(x, p)) = f(t, x, ζ(t, x, p)), (2.3a)
F_p(t,(x, p)) = $-$df/dx (t, x, ζ(t, x, p))†p $-$ dg/dx(t, x, ζ(t, x, p)) (2.3b)

1- We give ourselves an initial condition $p_0 \in R^d$ on the deputy state; by integrating the differential system (2.2), we obtain: p(T)$\in R^d$; this allows us to define the application:
F: $R^d \rightarrow R^d$, F (p_0) := p (T) -dh/dx (x(T)). (2.4)

2- We are looking for $p_0 \in R^d$ such that F(p_0)=0; it is a system of "d" nonlinear equations coupled in R^d, which we can (try to) solve by Newton's method.

References

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