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Relatively elementary definability of the class of universal graphic automata in the class of semigroups

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Graphic automaton is an automaton $A = (X, S, \delta)$, for which the set of states is equipped with the structure of a graph $G = (X, \rho)$ preserved by transition functions of the automaton. Graphic automaton $\mathrm{Atm}(G) = (G, \mathrm{End}\ G, \delta)$, where $\delta(x, \varphi) = \varphi(x)$ for $x \in X,\ \varphi \in \mathrm{End}\ G$, is a universally attracted object in the category of automata [1]. For this reason the automaton $\mathrm{Atm}(G)$ is called universal graphic automaton over the graph G. An edge $(x, y) \in \rho$ of the graph G is called proper if $(y, x) \notin \rho$. A graph is called quasi-acyclic if all its proper edges don't belong to any cycle. A quasi-acyclic graph having no proper edges in it is called a trivial quasi-acyclic graph, and non-trivial, otherwise. We investigate the problem of relatively elementary definability [2] of the class **Atm** of all universal graphic automata in the class **Sem** of all semigroups. The solution method consists in constructing an isomorphic copy of initial automaton $\mathbf{A} \in \mathbf{Atm}$ in its input symbol semigroup $\mathrm{Inp}(\mathbf{A})$ by means of narrow predicate calculus of the elementary semigroup theory and some fixed elements of $\mathrm{Inp}(\mathbf{A})$.

A mapping $c_x : X \to \{x\}$ is called constant mapping of a set X to an element $x \in X$. A set $\overline{X} = \{c_a : a \in X\}$ of all constant mappings in the semigroup $S = \mathrm{Inp}(A)$ is defined by the following formula of elementary semigroup theory: $M(x) = (\forall y)(yx = x)$.

Theorem. *There are formulas $M(x)$, $D(x, y, z)$, $P(u, v, x, y)$ in the language of elementary semigroup theory L_S such that for any universal graphic automaton $A = Atm(G)$ over non-trivial quasi-acyclic reflexive graph $G = (X, \rho)$ and its input symbol semigroup $S = End\ G$ the following conditions are satisfied:*

(1) *the set $\overline{X} = \{x \in S : M(x)\}$ is not empty;*

(2) *the formula $D(x, y, z)$ defines a ternary relation $\overline{\delta} \subset \overline{X} \times S \times \overline{X}$, satisfying the following condition: $(x, y, z_1), (x, y, z_2) \in \overline{\delta} \Longrightarrow z_1 = z_2$;*

(3) *there are elements x_0, y_0 in the input symbol semigroup S, such that the formula $P(x_0, y_0, x, y)$ defines an adjunct relation ρ on the set $\overline{X}$, so that a graphic automaton $\overline{A} = (\overline{G}, S, \overline{\delta})$ over the graph $\overline{G} = (\overline{X}, \rho)$ is isomorphic to universal graphic automaton $A = Atm(G)$.*

The obtained results make it possible to investigate the relationship between the basic properties of universal graphic automata and their input symbol semigroups, the interrelation of important properties of the elementary theories of classes of graphic automata and the elementary theories of classes of semigroups, for example, the problem of the elementary definability of universal graphic automata by their semigroups of input signals, etc.

References

1) Plotkin B. I., Greenglaz L. Ja., Gvaramija A. A. Elements of the algebraic theory of automata. *Moscow, Vyssh. Shkola*, 1994. 191 p. (in Russian).

2) Ershov Yu. L. Problems of decidability and constructive models. *Moscow, Nauka*, 1980. 416 p. (in Russian).