**Geometry of Quantum Mechanical Tunneling: Solution to the Navier-Stokes equation**

***Singh R.A.***

Project Assistant

Electrical: Modelling, Computation and Control (E-MC2) lab

Veermata Jijabai Technological Institute, Mumbai, India

rasingh@ee.vjti.ac.in

The research aims to define gravitational force at the planck scale. With this purpose, the phenomenon of quantum tunneling was investigated and a solution of the Navier-Stokes equation was developed[1].

The activity of matter-wave across a rectangular potential barrier was studied in the tunneling experiment. This phenomenon is governed by the Schrodinger wave equation which is the back-bone of the modern wave-mechanical theory[3], and has stayed inexplicable by Classical Mechanics theory, which defines gravitational field at the macroscopic scale[3]. The Navier-Stokes equation is chosen as the mathematical basis of this research.

Geometrical model for the action of matter-wave developed describes any material element as a smooth manifold with a Riemannian metric tensor that has three contravariant indices and one covariant index. The components of the tensor are related by a function,

 $f\left(x\right)=\left(\sqrt{nx}-\sqrt{x}\right)^{2}$ -(1)

Which, in the case when n = 0 or 4, and the coefficient of x = 1, results in the value of x. Essentially f(x) = 1

The derivative of this function, with respect to x,

 $f`\left(x\right)=-2\sqrt{nx}/\sqrt{x}+n+1$ -(2)

 $f`\left(x\right)=-2\sqrt{n}+n+1$ -(3)

When n = 0 or 4, eq. (3) results in f`(x) = 1

We hereby have,

 $f\left(x\right)=f`\left(x\right)$ -(4)

Or, $f\left(x\right)-f`\left(x\right)=0$ -(5)

Eq. (5) is congruent to the continuity form of the Navier-Stokes equation

 $∇⋅v=0$ [4]

The research thus derives a solution to the Navier-Stokes equation as the Riemannian metric tensor defined above, which is a smooth function[2]. The study establishes the Tunneling phenomenon as synonymous with the concept of collisions studied in Classical Mechanics. The deviation in the time scale for the macroscopic phenomenon is minuscule and non-inferential, which had generally kept the similarities of the cases hidden. Any kind of deformation in space and time is a result of energy differences in the system under consideration. With this learning, the research put forth the definition of the gravitational curvature at the planck scale.

Literature

1. Rangamani, M. (2009) 'Gravity and hydrodynamics: lectures on the fluid-gravity correspondence', Classical and Quantum Gravity, 26(22), pp. 224003.
2. Fefferman, C. L. (2022) Navier-Stokes Equation [online]. Clay Mathematics Institute.

Available from:https://www.claymath.org/wp-content/uploads/2022/06/navierstokes.pdf [Accessed: 9 March 2025]

1. University of Illinois Urbana-Champaign (2015) Tunneling [online]. Available from: https://courses.physics.illinois.edu/phys485/fa2015/web/tunneling.pdf [Accessed: 9 March 2025]
2. Wiedemann, E. (2018) Navier-Stokes Equations [online]. University of Ulm. Available from:https://www.uni-ulm.de/fileadmin/website\_uni\_ulm/mawi.inst.020/wiedemann/Skripte/EW\_Navier-Stokes\_Equations.pdf [Accessed: 9 March 2025]