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Calibration of Geometric Brownian Motion Stock Prices with Stochastic Interest Rates: A GBM-CIR Framework

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Abstract

This paper extends the classical Geometric Brownian Motion (GBM) model by integrating stochastic interest rates modeled via the Cox-Ingersoll-Ross (CIR) process. This GBM-CIR framework captures real-world financial dynamics where interest rates fluctuate and are correlated with stock prices. We employ finite difference schemes, Monte Carlo simulations, and machine learning-based calibration to estimate model parameters. Empirical validation using real market data demonstrates that our approach improves predictive accuracy over traditional models.

1. Introduction

Stock price modeling is a core aspect of quantitative finance, with the Black-Scholes model serving as a foundation. However, assuming a constant risk-free rate oversimplifies market dynamics. We enhance the GBM framework by incorporating a CIR-driven stochastic interest rate process and calibrate it using real data.

2. Model Framework

The stock price process S_t follows a GBM:

$$dS_t = R_t S_t dt + \sigma S_t dW_t^S,$$

and the risk-free rate R_t follows a CIR process:

$$dR_t = (a - bR_t)dt + v\sqrt{R_t}dW_t^R.$$

Brownian motions W_t^S and W_t^R are correlated with coefficient ρ. We discretize these SDEs using an Euler-Maruyama scheme for efficient numerical implementation.

3. Calibration and Numerical Methods

We estimate parameters by minimizing a loss function using historical stock prices and interest rates. Monte Carlo simulations generate synthetic paths, and a stopping rule prevents overfitting. The model is implemented in Python with object-oriented programming for efficiency.

4. Empirical Results

Applying our model to MTSS stock and RUONIA rates, we observe lower RMSE and MAE compared to a baseline GBM. The inclusion of stochastic rates enhances predictive performance, capturing market fluctuations more effectively.

5. Conclusion and Future Research

Our findings confirm that stochastic interest rates improve stock price modeling accuracy. The GBM-CIR framework can be extended to derivative pricing, portfolio optimization, and risk management. Future work may explore stochastic volatility extensions, such as the Heston model, and alternative calibration techniques.

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